CHAPTER 6. STOCHASTIC OPTIMAL CONTROL APPROACH

In this chapter, the stochastic optimal control method employed to solve the stochastic formation mission design problem is introduced. First, the main sources of uncertainty in air traffic management are described. Later, the formulation of the stochastic switched optimal control problem is introduced. After that, the methodology to convert the stochastic constraints into deterministic constraints and the stochastic objective functional into a deterministic objective functional is described. This methodology is based on nonintrusive generalized polynomial chaos stochastic collocation. Finally, the method employed to conduct the sensitivity analysis is described, the aim of which is to identify the random variables that have more influence on the variability of a component of the solution of the stochastic formation mission design problem.

6.1. Introduction

An OCP of a stochastic switched dynamical system is an OCP in which the continuous dynamics of the system is represented by stochastic differential equations, the objective functional is a stochastic functional, and the constraints, which are defined by means of stochastic functions, must be satisfied almost surely, i.e., with probability 1. The set of possible exceptions in which the constraints are not satisfied may be non-empty but must have probability 0. Therefore, in this thesis, the adjective stochastic means that the functions and the solutions of the differential equations depend on a vector of random variables. This problem is referred to as the stochastic switched optimal control problem (SSOCP).

Modeling, control, and optimal control of stochastic switched systems are addressed in [Alwan and Liu, 2018, Zhu, 2019], in which the random factors acting on the continuous dynamic models of the switched system in each discrete state are represented by some idealized processes such as the Wiener process, and tools such as stochastic calculus have been employed to obtain solutions.

Another approach to model uncertainties in switched dynamical systems is to treat uncertainties as random variables or random processes and recast the original deterministic switched dynamical system as a stochastic switched dynamical system. This type of stochastic systems are different from those represented by classical stochastic differential equations, where the random inputs are idealized processes. Consider a stochastic optimal control problem in which only random variables are present in its formulation like in the formation mission design problem studied in this thesis. One of the most commonly used methods to solve OCPs of stochastic dynamical systems of this type is the Monte Carlo sampling [Shapiro, 2003], in which independent realizations of the random variables are generated based on their probability distributions. For each realization, the OCP becomes deterministic. After solving the deterministic instances of the problem that correspond to these realizations of the random variables, an ensemble of solutions is obtained, from which statistical information can be extracted. Although Monte Carlo sampling is straightforward to apply as it only requires iterative resolutions of deterministic instances of the OCP, it requires a large number of solutions, because the solution statistics converge relatively slowly. For example, the mean value typically converges as $1/\sqrt{N}$, where N is the number of realizations. The need for large number of solutions for accurate results can lead to an excessive computational cost, especially for OCPs that are already computationally intensive in the deterministic settings, as the mission design problem considered in this thesis. The Generalized Polynomial Chaos (gPC) expansion can contribute to alleviating this drawback.

A systematic and coherent presentation of numerical strategies for uncertainty quantification and stochastic computing is given in [Xiu, 2010], with a focus on the methods based on the gPC approach. Both the intrusive stochastic Galerkin and the nonintrusive stochastic collocation approaches to gPC are illustrated in detail. The nonintrusive stochastic collocation approach based on regression is described in [Sudret, 2008] and [Blatman and Sudret, 2010].

In this thesis, a stochastic collocation method has been selected. With this approach, a small number of sample points of the random variables are used to jointly solve particular instances of the SSOCP. The obtained solutions are then expressed as orthogonal polynomial expansions in terms of the random variables using these sample points. This is a nonintrusive methodology because the model equations are not altered. Depending on the distributions of the random variables, different types of orthogonal polynomials can be chosen to achieve better numerical precision. This technique allows statistical and sensitivity analysis of the stochastic solutions to be conducted at a low computational cost. Thus, the gPC method converts the SSOCP into an augmented deterministic SOCP, in which particular instances of the SSOCP are solved together as a single deterministic OCP. It is important to point out that the gPC method is applicable to solve the SSOCP when the solution depends smoothly on the random variables. This means that the solutions obtained for each combination of sample points of the random variables must give rise to the same discrete solution, i.e., to the same sequence of discrete states of the switched dynamical system that represents the formation mission.

Sources of uncertainty in ATM

In [Cook et al., 2015], the different sources of uncertainty that affect ATM have been classified according to the following five major groups:

- Data uncertainty. This type of uncertainty is due to the presence of some level of uncertainty in the data or some degree of inaccuracy in the models.

- Data unavailability. In this case, there is lack of information due to managerial or technological barriers.

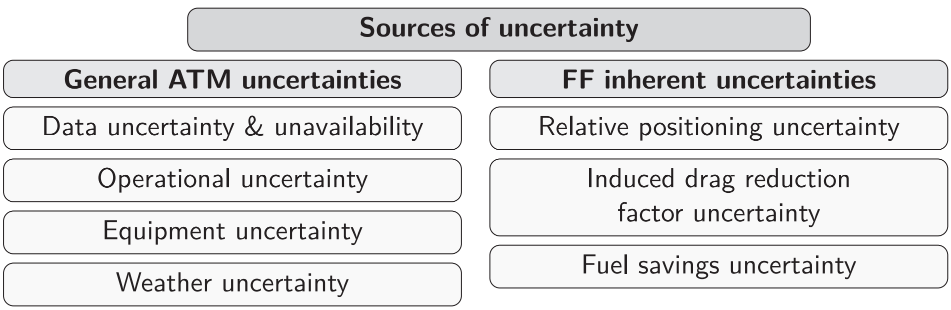
- Operational uncertainty. In this case, uncertainty is due to decisions taken by humans, e.g. air traffic controllers, flight dispatchers, and pilots.

- Equipment uncertainty. This type of uncertainty is due to problems with aircraft or with the communications, navigation and surveillance equipment.

- Weather uncertainty. This type of uncertainty is due to the presence of winds, thunderstorms, snowfalls, and fog.

From all this sources of uncertainty, it is well known that uncertainty in flight departure times is among the main causes of trajectory uncertainty, which generates inefficiency in the ATM system [Rivas and Vazquez, 2016]. Timing is not only crucial in general ATM operations, but also in formation missions. Indeed, considering the usual cruise speed of most long-haul commercial aircraft, missing the rendezvous location by, for instance, ten minutes means spatially missing the partner aircraft by 150 km. Such cases require catch-up maneuvers, which result in a loss of performance compared to the planned formation mission.

Fig. 6.1. Sources of uncertainty affecting formation flight.



Additionally to the former general ATM-related uncertainties, there are uncertainties inherent to formation flight.

As seen in Chapter 2, in extended formations, due to the great distance between the leader and the follower aircraft, instabilities such as meandering and external factors, may affect the motion of the vortices. It is therefore important to maintain the relative position between the follower aircraft and the leader's wake vortices precisely, as the potential formation benefits are very sensitive to that relative positioning. However, the relative positioning between the follower aircraft and the leader's wake vortices can not be kept precisely. Therefore, the induced drag reduction factor and the fuel savings achieved in formation flight are actually uncertainties of the formation mission design problem. Fig. 6.1 gives a schematic overview of both, general uncertainties related to ATM and uncertainties inherent to formation flight.

A great number of research studies focused on stochastic modeling. In [Shone et al., 2021] a review of the literature on stochastic modeling with applications to ATM is provided, include literature on stochastic optimal control.

Most of these studies focused on the conflict detection and resolution problem. In [Hu et al., 2005], the problem of aircraft conflict prediction is studied for two-aircraft midair encounters. First, a model is presented for prediction of the aircraft positions along a time horizon, during which each aircraft is following a prescribed flight plan in the presence of additive wind perturbations on its velocity. Then, a method for estimating the probability of conflict is proposed. This method is based on a Markov chain approximation of the stochastic processes that models the aircraft flight. In [Prandini and Hu, 2009], the aircraft conflict prediction problem is formulated as a reachability problem in a stochastic hybrid system framework. Specifically, a switching diffusion model is employed to predict the future positions of an aircraft following a given flight plan, and the probability that the aircraft enters an unsafe region of the airspace is estimated using a numerical algorithm for reachability computation.

In [Li et al., 2014], an approach to aircraft trajectory optimization in the presence of uncertainties based on gPC is presented, where only one random variable has been considered, which represents an uncertain aerodynamic parameter of the dynamic model of the aircraft. This random variable has been modeled as a uniformly distributed random variable. In [Matsuno et al., 2015], a stochastic optimal control method based on gPC is developed for determining conflict-free aircraft trajectories under wind uncertainty. The random processes that represent the components of the wind speed are approximated as a linear combination of deterministic functions multiplied by independent random variables using the Karhunen-Loève expansion.

Although there is a growing interest in addressing uncertainties in the field of ATM, there are very few studies that study formation flight for commercial aircraft in the presence of uncertainties. One of these few research studies is [Kent and Richards, 2014], in which the impact of ground delays on formation flight is studied using stochastic dynamic programming.

Specification of the stochastic formation mission design problem

In this thesis, the formation mission design problem for commercial aircraft in the presence of uncertainties in some of the parameters and boundary conditions of the problem is studied. Given several commercial flights, the stochastic formation mission design problem consists in establishing how to organize them in formation or solo flights and in finding the trajectories that minimize the expected value of the DOC of the formation mission in the presence of uncertainties. Since each aircraft can fly solo or in various positions within a formation, the mission is modeled as a stochastic switched dynamical system, in which the flight modes of the aircraft are described by sets of stochastic ordinary differential equations, the discrete state describes the combination of flight modes of the individual aircraft, and logical constraints establish the switching logic among the discrete states of the system. Stochastic switched dynamical systems inherit all the features of the deterministic switched dynamical systems seen in Chapter 4.

Both the discrete and the continuous dynamics are affected by uncertainties. This general definition can encompass a wide range of stochastic phenomena. In this thesis, only the stochastic hybrid systems in which random variables only affect the continuous dynamics have been considered. Specifically, random variables represent uncertain in the departure times of the aircraft and in the fuel burn savings for the trailing aircraft. The probability distribution functions of these random variables are assumed to be known.

In the following section, the formulation of an OCP of a stochastic switched dynamical system is introduced.

6.2. The stochastic switched optimal control problem

The presence of random variables in the SOCP converts it into an SSOCP. In the formation mission design problem studied in this thesis, not all the elements of the SSOCP contain random variables. Specifically, the fuel consumption reduction factor $\mathfrak{R}_{\text{fuel}}$ in the mass flow rate equation (3.14) of the aircraft flying in formation as a trailing or an intermediate aircraft is a random variable. In contrast, the equations of motion associated with the state variables φ, λ, χ, and V do not contain random variables. The departure times of the aircraft are also assumed to be random variables of the SSOCP. One of these departure times coincides with *ti*, the initial time of the formation mission. These random variables are assumed to be independent and characterized by probability density functions.

Let $\theta = (\theta_1, \theta_2, \ldots, \theta_N)$ be the vector of random variables that represent the random parameters of the SOCP. For sake of clarity, hereinafter the subindex "SS" will be employed in the formulation of the OCP for stochastic switched dynamical systems. It is assumed that the control vector $u_{ss}(t)$ is deterministic and, therefore, it is a function of time only, whereas the state vector $x_{ss}(t, \theta)$ is stochastic and, therefore, it is a function of both time and the vector of random variables, θ [Li et al., 2014].

The continuous-time SSOCP has been formulated as in [Li et al., 2014], namely all the elements of the problem depend on θ and the dynamic equations, the path constraints, and the boundary conditions must be satisfied almost surely. In the formulation of the SSOCP, the dynamical equations and endpoint constraints are:

$$\begin{aligned}
&\dot{x}_{SS}(t,\theta) = f_{SS,v_{SS}(t)}\left(t, x_{SS}(t,\theta), u_{SS}(t), \theta\right) \text{a.s.},\\
&x_{ss}(t_I,\theta) = x_I \in \mathbb{R}^n \text{ a.s.},\\
&x_{SS}(t_F,\theta) = x_F \in \mathbb{R}^n \text{ a.s.},\\
&v_{SS}(t) \in \{0,1\}, t_I \le t \le t_F.
\end{aligned} \tag{6.1}$$

The performance index of the SSOCP has the following form:

$$\begin{aligned}
&J_{SS}\left(t, x_{SS}(t,\theta), u_{SS}(t), v_{SS}(t), \theta\right) =\\
&M_{SS}\left(t_F, x_F, \theta\right) + \int_{t_I}^{t_F} L_{SS,v_{SS}(t)}\left(t, x_{SS}(t,\theta), u_{SS}(t), \theta\right) dt.
\end{aligned} \tag{6.2}$$

The SSOCP is thus stated as follows

$$\min_{u_{SS} \in \Omega, v_{SS} \in \{0,1\}} J_{SS}\left(t, x_{SS}(t,\theta), u_{SS}(t), v_{SS}(t), \theta\right), \tag{6.3}$$

subject to dynamical equations and endpoint constraints from the set of Eqs. (6.1). It can be seen that the difference between the formulations of the deterministic SOCP and the SSOCP is that the vector random variables θ is included in the latter.

It is important to point out that probabilistic constraints, also known as chance constraints, could be introduced in the formulation of the SSOCP [Matsuno et al., 2015]. However, in this thesis, chance constraints have not been considered in the formulation of the SSOCP employed to solve the stochastic formation mission design problem.

6.2.1. The generalized polynomial chaos expansion

In this section, following [Li et al., 2014, Matsuno et al., 2015], the gPC expansion is introduced and the method for determining the stochastic solution of the SSOCP and computing its statistical information is described.

Using the gPC expansion, the stochastic solution of the SSOCP can be approximated by a finite sum of orthogonal polynomials. The P-th order gPC approximation of the stochastic solution of the SSOCP, $z(t, \theta)$, can be expressed as

$$z_P(t,\theta) = \sum_{m=1}^{M} C_m(t)\Phi_m(\theta), \tag{6.4}$$

where $\theta = (\theta_1, \theta_2, \ldots, \theta_N)$ is a vector of independent random variables, $C_m(t)$ are the corresponding coefficients of the expansion, which can be obtained using either a nonintrusive or an intrusive approach, and $\phi_m(\theta)$ are the multivariate orthogonal polynomial basis functions, which are calculated from the l_i-th order one-dimensional polynomial basis function $\phi^{(li)}(\theta_i)$ of the random variable θ_i by means of the tensor product rule as follows:

$$\Phi_m(\theta) = \prod_{i=1}^{N} \theta_i^{(l_i)}(\theta_i). \tag{6.5}$$

Notice that, in Eq. (6.5), a unique combination of $l_i, i = 1, 2, \ldots, N,$ corresponds to each subscript m, which satisfy the condition $\Sigma_{i=1}^{N} l_i \leq P.$ The number of tensor product basis functions is $M = \begin{pmatrix} N+P \\ N \end{pmatrix}$.

Orthogonal polynomials basis functions

The orthonormal polynomials in Eq. (6.5) satisfy the following orthogonality condition

$$E\left[\phi_i^{(j)}(\theta_i)\phi_i^{(k)}(\theta_i)\right] = \int \phi_i^{(j)}(\theta_i)\phi_i^{(k)}(\theta_i)\rho_i(\theta_i)d\theta_i = \delta_{jk}, \tag{6.6}$$

where E is the expected value operator, $\rho_i(\theta_i)$ is the probability density function (PDF) of the random variable θ_i, and δ_{jk} is the Kronecker delta function.

To improve convergence, the choice of the orthogonal polynomials should be made on the basis PDF $\rho_i(\theta_i)$. For instance, the Legendre polynomials are the best choice for uniform random variables, whereas Hermite polynomials are the best option for Gaussian random variables. Some relationships between continuous and discrete probability distributions and their correspondent gPC basis polynomials are shown in Tables 6.1 and 6.2, respectively.

Table 6.1. Correspondence between continuous random variables type and the gPC basis polynomial

Distribution	gPC basis polynomial
Gaussian	Hermite
Uniform	Legendre
Gamma	Laguerre

Table 6.2. Correspondence between discrete random variables type and the gPC basis polynomial.

Distribution	gPC basis polynomial
Poisson	Charlier
Binomial	Krawtchouk
Hypergeometric	Hahn

6.2.2. The stochastic collocation approach

The expansion coeficientes, $C_m(t)$, can be obtained using intrusive or nonintrusive approaches.

In the stochastic Galerkin method, the gPC expansions in terms of the random variables that represent the sources of uncertainty such as parameters and boundary conditions, are first determined and then the gPC expansions are substituted into the model equations. The Galerkin projection is then performed to project the model equations onto the random space spanned by the polynomial basis. This projection transforms each stochastic equation of the model into a set of coupled deterministic equations. Finally, the resulting system of equations is solved with a suitable numerical method. This is an intrusive methodology in the sense that the model equations are altered. The stochastic Galerkin method can be challenging when the stochastic model equations take complicated forms. In this case, the derivation of the set of deterministic equations can be very difficult or even impossible [Li et al., 2014].

Within the nonintrusive methods, the regression approach and the stochastic collocation method can be employed. The nonintrusive stochastic collocation approach based on regression is described in [Sudret, 2008] and [Blatman and Sudret, 2010]. This method is specially appropriate when the number of random variables considered is high, in general, larger than 10 variables [Matsuno et al., 2015]. In stochastic collocation methods the stochastic model equations are satisfied at a discrete set of points, called nodes, in the corresponding random space. From this point of view, all classical sampling methods like Monte Carlo sampling are collocation methods. However, in stochastic collocation, polynomial approximation theory is employed to locate the nodes, known as collocation points, strategically to increase the numerical accuracy and significantly reduce the sample points.

In this thesis, the coefficients $C_m(t)$ of the expansion (6.4) are calculated using a nonintrusive gPC based stochastic collocation method as follows

$$C_m = E\left[z(t,\theta)\Phi_m(\theta)\right] = \int z(t,\theta)\Phi_m(\theta)_\rho(\theta)\,d\theta, \tag{6.7}$$

where $\rho(\theta) = \prod_{i=1}^{N} \rho_i(\theta_i)$ is the joint probability density function of the vector of random variables θ.

A Gaussian quadrature can be employed to approximate the integral in Eq. (6.7) using a set of q collocation points $\theta^{(j)}$ for each random variable θ_i and calculating the corresponding set of quadrature weights $\alpha_i^{(j)}$, $j = 1,2,\ldots,q$.

Higher precision in the quadrature can be achieved by increasing the number of collocation points q. Using the tensor product rule, the total number of collocation points is q^N. This number, which could become large when N, the number of random variables in θ, grows, can be reduced to Q using the sparse grid quadrature based on the Smolyak rule [Xiu, 2010].

Let $\theta^{(j)}$ be the collocation points and $\alpha^{(j)}$, $j = 1,\ldots,Q$, the corresponding weights. Then, the coefficients of the expansion C_m in Eq. (6.7) are approximated by:

$$C_m \approx \sum_{j=1}^{Q} z\left(t,\theta^{(j)}\right)\Phi_m\left(\theta^{(j)}\right)\alpha^{(j)}, \tag{6.8}$$

where $z(t, \theta^{(j)})$ denotes the solution of the augmented SOCP obtained using the j-th collocation point $\theta^{(j)}$ and $\Phi_m(\theta^{(j)})$ represents the multivariate orthogonal polynomial basis function evaluated at the j-th collocation point $\theta^{(j)}$. Thus, the SSOCP is converted into an augmented deterministic SOCP, in which particular instances of the SSOCP, which correspond to the collocation points of the random variables, are combined into a single optimal control problem.

The resulting augmented SOCP is solved by means of the numerical methods described in chapters 4 and 5. The expected value and variance of the stochastic solution $z_P(t, \theta)$ are calculated using the coefficients of the expansion as follows:

$$E\left[z_P\left(t,\theta\right)\right]=C_1\left(t\right), \tag{6.9}$$

$$\mathrm{VAR}\left[z_P\left(t,\theta\right)\right]=\sum_{m=2}^{M}C_m^2\left(t\right), \tag{6.10}$$

where VAR denotes the variance operator. As stated in Eqs. (6.9) and (6.10), the expected value coincides with the first coefficient, whereas the variance is the sum of the squares of the other coefficients of the gPC expansion. Using the former expressions, statistical information of the solutions of the SSOCP can be easily computed. Additionally, the mean and the standard deviation of some component of the stochastic solution can be expressed as a function of the variables of the SSOCP and included in the objective functional and the constraints of the problem.

A complete treatment of the gPC expansion can be found in the monography [Xiu, 2010], whereas the procedure followed to determine the stochastic solution of the SSOCP and to compute its statistical information is schematically represented in Fig. 6.2. Further details can be found in [Li et al., 2014, Matsuno et al., 2015].

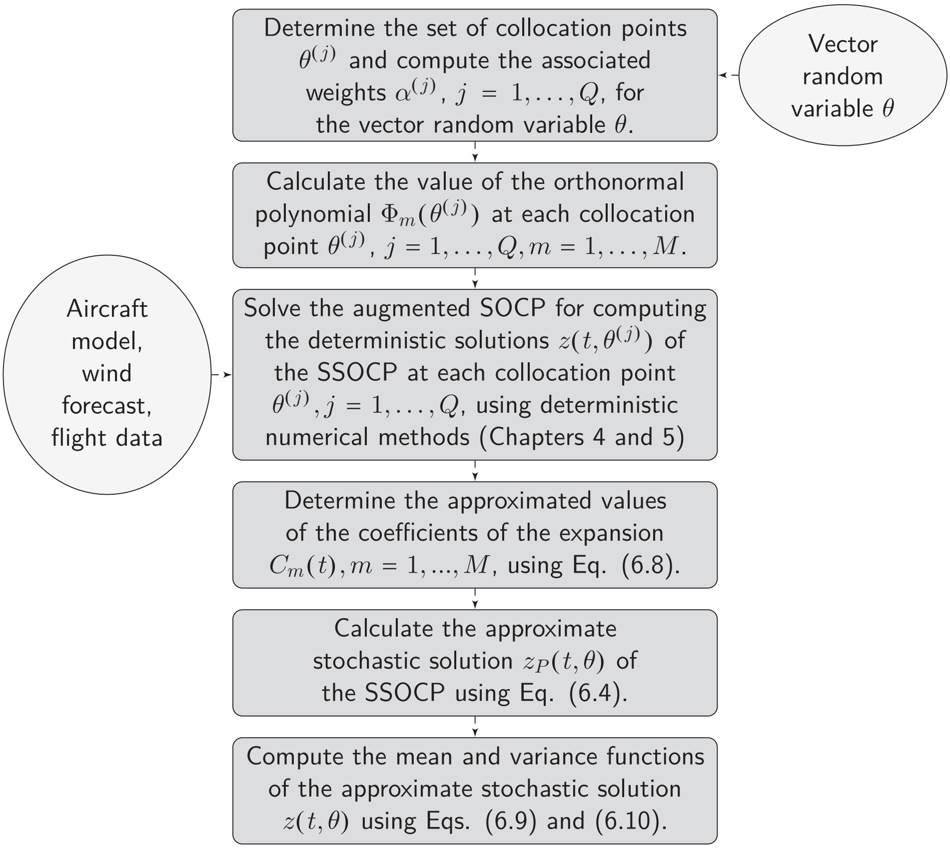
6.2.3. Specification of the stochastic switched optimal control problem

Following the technique described in [Li et al., 2014], the stochastic constraints from Eqs. (6.1) have been converted into deterministic constraints. Likewise, the stochastic objective functional of the SSOCP from Eq. (6.2) has been converted into a deterministic functional by taking its expected value. The new formulation of the SSOCP can be expressed as follows

$$
\begin{aligned}
&\min_{\substack{u_{SS}\in\Omega,\\ v_{SS}\in\{0,1\}}} J_{SS} = E\left[M_{SS}\left(t_F, x_F, \theta\right) + \int_{t_I}^{t_F} L_{SS,v_{SS(t)}}\left(t, x_{SS}\left(t,\theta\right), u_{SS}\left(t\right), \theta\right) dt \right] \\
&s.t. \\
&\dot{x}_{1SS}\left(t,\theta_1\right) = f_{SS,v_{SS}(t)}\left(t, x_{SS}\left(t,\theta_1\right), u_{SS}\left(t\right), \theta^{(1)}\right), \\
&\dot{x}_{2SS}\left(t,\theta_2\right) = f_{SS,v_{SS}(t)}\left(t, x_{SS}\left(t,\theta_2\right), u_{SS}\left(t\right), \theta^{(2)}\right), \\
&\vdots \\
&\dot{x}_{N_{SS}}\left(t,\theta_N\right) = f_{SS,v_{SS}(t)}\left(t, x_{SS}\left(t,\theta_N\right), u_{SS}\left(t\right), \theta^{(Q)}\right), \\
&E\left[x_{SS}\left(t_I,\theta\right)\right] = x_I \in \mathbb{R}^n, \\
&E\left[x_{SS}\left(t_F,\theta\right)\right] = x_F \in \mathbb{R}^n, \\
&v_{SS}\left(t\right) \in \{0,1\}, t_I \le t \le t_{F,}
\end{aligned}
\tag{6.11}
$$

where $\theta^{(1)}$, $\theta^{(2)}$,..., $\theta^{(Q)}$ are the Q collocation points of the vector random variable θ.

Fig. 6.2. Schematic diagram of the procedure for determining the stochastic solution of the SSOCP and computing its statistical information.



Solution of the stochastic switched optimal control problem

As already seen in Chapter 4, deterministic switched dynamical systems are described by both a continuous and a discrete dynamics, in which the transitions among discrete states are not established in advance. Stochastic switched dynamical systems inherit all the features of the deterministic switched dynamical systems.

The stochastic collocation method converts the SSOCP into an augmented deterministic SOCP. With this approach, a small number of sample points of the random parameters are employed to jointly solve particular instances of the SOCP. The same methodology as the one explained in Chapter 4 for deterministic SOCP can be followed, obtaining an augmented deterministic

EOCP with inequality and equality constraints which can be solved using the optimal control techniques explained in Chapter 5.

The solution of the SSOCP includes both the state and control variables of the system that represent the formation mission. Due to the presence of the vector of random variables in the SSOCP, the solution is stochastic, i.e., it depends on the actual realization of the vector random variable. In particular, it is a random function of state or time. Therefore, the solution is actually a random process, characterized by the mean function and the corresponding 95% confidence envelope. Specifically, the observed values of a state variable at a given time t is a random variable. In this case, the random process is a random function of time. Likewise, the time at which a given value of the state variable is reached is a random variable. This means that the timing of the trajectories, which is defined as the time at which a state is reached, is also a random process. In this case, the random process is a random function of the state. In this paper, the timing of the trajectories is represented as a random function of the orthodromic distance from the departure location. For the sake of ease of exposition, the stochastic solution of the SSOCP is denoted in the following sections as a random function of time. All the random variables observed at specific time instants or states are characterized by their expected values and by the corresponding 95% confidence intervals, including the arrival times and the fuel consumption of the aircraft of the formation mission.

6.3. Sensitivity analysis of the solution

The purpose of the sensitivity analysis is to identify the random variables that have more influence on the variability of a component of the stochastic solution with the aim of reducing its variability acting on the source. Therefore, in this thesis, a global sensitivity analysis of the stochastic solution is also conducted [Saltelli et al., 2008] in the numerical experiment that involves two random variables such as Experiment B presented in Chapter 8, in which a two-aircraft transoceanic mission design problem with uncertainty in the departure times of both flights is addressed.

In particular, the aim of the variance-based sensitivity analysis is to quantify what proportion of the variance of each component of the solution $z(t, \theta)$ of the formation mission design problem is due to the variance of each component $\theta_j, j = 1,2,\ldots,N$ of the random vector θ.

In this thesis, this analysis relies on the computation of the so-called Sobol' indices, under the assumption that the random variables in θ are independent [Saltelli et al., 2008]. Sobol' indices can be directly derived from the coefficients $C_m(t)$ of the gPC expansion (6.4) as explained in [Trucchia et al., 2019].

Following [Trucchia et al., 2019], the Sobol' index $S_{i,j}(t)$ for the i-th component of the solution $z(t, \theta)$ and the j-th component of the random variable θ can be expressed as

$$S_{i,j}(t) = \frac{1}{\mathrm{VAR}_i} \sum_{m \in I} C_m^2(t), \tag{6.12}$$

where VAR_i is the variance calculated in Eq.(6.10) for the i-th component of the solution and I is the set of multi-indices such that the computation of $S_{i,j}(t)$ only includes terms that depend on the random variable θ_j, $j = 1,2 ,\ldots,N$.

6.4. Conclusions

In this chapter, the formulation of the stochastic optimal control problem employed to model the stochastic formation mission design problem is presented and the approach followed to solve it is described. It is a nonintrusive polynomial chaos based stochastic collocation method, which converts the stochastic switched optimal control problem into an augmented deterministic switched optimal control problem. With this approach, a small number of sample points of the random parameters are used to jointly solve particular instances of the switched optimal control problem. The obtained solutions are then expressed as orthogonal polynomial expansions in terms of the random parameters using these sample points. The technique allows statistical and global sensitivity analysis of the stochastic solutions to be conducted at a low computational cost.